Kuwait University
Department of Mathematics
& Computer Sciences

Math.101
First Exam.
(Incomplete)

November 13th, 1995.
Duration: 75 minutes

Calculators are not allowed

Answer the following questions:

1. (a) Evaluate the following limits, if it exists,

$$(i)\ \lim_{x\to 1}\frac{x+\sqrt{8x+1}-4}{x-1}, \qquad (ii)\ \lim_{x\to 0}\frac{\sin 2x-2\sin x}{x^2}.$$

(b) Let

$$x-5\le f(x)\le \sin x-g(x)\ .$$

If $g(x)$ is continuous at $x=0$ and $g(0)=5$, then evaluate $\lim_{x\to 0} f(x)$.

2. (a) Find the vertical and horizontal asymptotes for the graph of f, if any, where

$$f(x)=\frac{|\sqrt{x}-2|}{x-4}.$$

(b) Use the intermediate value theorem to show that the function

$$f(x)=\sin^3 x-\cos x$$

has at least one a real zero.

3. (a) let

$$f(x)=\begin{cases} A\cos x+B\sin x & \text{if } x\ge 0,\\ Ax^2-x+3B & \text{if } x<0.\end{cases}$$

Find the constants A and B for which $f'(0)$ exists.

(b) Use the definition of the derivative to find $f'(x)$, where

$$f(x)=\frac{1}{x-1}\ \text{, where } x\neq 1.$$

4. (a) Let

$$f(x)=ax^{\frac{1}{5}}+b$$

where a and b are constants. Show that the graph of $f(x)$ has a vertical tangent at the point $(0,b)$.

(b) Find an equation of the tangent line to the graph of the function

$$f(x)=\sec x\tan x+1$$

at $x=0$.

(Good Luck)